Relaying-Assisted Multiuser Networks in FBL Regime: Achievable Reliability-Constrained Throughput

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***Abstract*—In this work, we study a multi-user relaying network operating with finite blocklength (FBL) codes, where a single relay node is responsible for relaying the data from a source node to multiple users. We consider two relaying principles, i.e., decode-and-forward (DF) and amplify-and-forward (AF) relaying, together with two downlink strategies for relaying to multiple users, i.e., broadcasting and time division multiple access (TDMA), which result in total in four combinations. Taking the fairness into account, we formulate optimization problems to maximize the minimum throughput among all users via blocklength allocation, while guaranteeing the reliability. We start with the combination strategy of TDMA-DF, where the DF relay serves multiple users in a TDMA manner in the second hop. To solve the nonconvex problem, we have constructed a convex approximation for the problem and accordingly proposed iterative algorithm, which is capable of iteratively improving the minimum throughput until a convergence to a suboptimal point. Afterwards, the iterative algorithm is then extended to other combinations, i.e., broadcasting-DF and TDMA-AF, while for broadcasting-AF, the optimal blocklength allocation can be directly derived out. Finally, via simulation results, we validate the convergence of our proposed iterative algorithms, and by comparison depict the benefits of DF relaying over AF relaying and the advantages of broadcasting and TDMA strategies in different scenarios.**

***Index Terms*—Relaying principles, multiple users, finite block length (FBL), time-division multiple access (TDMA).**

I. INTRODUCTION

In 5G and beyond wireless networks, ultra-reliable low-latency communications (URLLC) have enabled various applications, like health monitoring, remote operations, autonomous driving, virtual and augmented reality [1]. While guaranteeing the low-latency constraints, numerous researches have been performed to enhance the transmission reliability, for instance, via resource allocation [2] and retransmission scheme design [3]. In particular, given a maximum tolerant transmission error probability, the achievable reliability-constrained performance in URLLC networks has been investigated in [4]–[6].

To further strengthen the network functionality, researchers are motivated to implement copious advanced techniques in URLLC networks. Among them, relaying, under either a decode-and-forward (DF) principle or an amplify-and-forward (AF) principle, appears as a particularly promising technology, which has already shown to be capable of enlarging the network coverage [7] and enhancing the throughput [8]–[10]. While integrating relay in URLLC networks, performance improvement has also been observed with respect to transmission reliability [11], data rate [12] and energy efficiency [13].

In addition, URLLC networks are usually expected to connect massive users/devices, where the user scheduling policy plays an important role in determining the service quality and fairness. For a frequency-limited scenario, there are mainly two types of user scheduling strategies: i. time-division multiple access (TDMA) which is per-user scheduling via separated slots; ii. broadcasting via a shared long slot/blocklength. Note that due to the low-latency requirement, URLLC transmissions are required to operate with finite blocklength (FBL) codes, i.e., transmitting in a so-called FBL regime [14]. In such case, the transmission error probability cannot be ignored even setting the coding rate below the Shannon capacity [14], and is decreasing with respect to the blocklength. Hence, the tradeoff between the above two strategies are actually in the blocklength allocation. More specifically, with broadcasting strategy, data for all users can be encoded in a shared long blocklength, which can potentially reduce the decoding error probability. However, in broadcasting strategy, all users will decode the same packet and extract out their own data, which implies that the performance of broadcasting strategy will be limited by the bottleneck user with the worst channel. By contrast, TDMA strategy is capable of adaptively allocating blocklength among users but will also shorten the blocklength for each users. In the literature, with TDMA strategy, [15], [16] have studied the resource allocation respectively for error probability minimization and sum throughput maximization in multi-user URLLC networks, while the encoding process of broadcasting strategy has been examined in [17]. However, a comparison of the two strategies, as well as the tradeoff, has not been investigated yet.

Moreover, in a relaying-assisted mutli-user URLLC network, different relaying principles (DF or AF) will also affect both TDMA and broadcasting strategies differently. For instance, without decoding capability, an AF relay cannot change the multi-user serving strategies, so that both the strategy and blocklength allocation have to be kept the same for different relaying hops. With an additional decoding process, DF relay will be more flexible in deciding both the strategies and blocklength allocation on multiple relaying hops.

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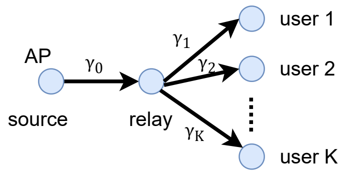

Fig. 1. Multi-user relaying network.

So far, to the best of our knowledge, a fundamental FBL performance characterization in relaying-assisted multi-user downlink network including reliability-constrained throughput analysis, together with a concrete study of different downlink strategies, is still missing in the literature. Therefore, in this work, we are motivated to study the multi-user relaying network within FBL regime. We take into account different relaying principles and different serving strategies for multiple users, i.e., broadcasting and TDMA. Under a reliability constraint, we build blocklength allocation problems for maximizing the minimum throughput among users, which are addressed via proposed iterative algorithms.

The remaining of this paper is organized as follows. We first introduce the system model in Section II. In Section III, we formulate the blocklength allocation problem under TDMA-DF strategy and proposed an iterative algorithm, which is extended to other strategies in Section IV. Finally, the work is evaluated in Section V and concluded in Section VI.

II. SYSTEM MODEL

In this paper, we consider a relaying network with multiple users, as depicted in Fig. 1. An access point (AP), i.e., the source node, is responsible for generating independent data to multiple users, while the data transmission is required to be completed under a latency requirement. For instance, in an IoT network with multiple devices as the users, the AP may need to send different control instructions or different necessary configuration messages to all the devices, which should be completed before a planned time point. Next, to compensate the large pathloss between the AP and multiple users, a relay node is deployed to assist the data transmission, while serving all the users. In particular, the relay node will first receive all the data from the AP and then forward them to the users.

We assume there are in total K users to be served. The channel gain between the AP and relay is denoted by h_0, while the channel gain between relay and the user k is given by h_k, $k \in \{1,...,K\} \triangleq \mathcal{K}$. We denote by P_1 and P_2 the constant transmit power at AP and relay, respectively. We assume all the communication channels are noisy channels with additive white Gaussian noise (AWGN). The noise power for the channel between AP and relay is denoted by σ_0^2, while the noise power between relay and user $k \in \mathcal{K}$ is given by σ_k^2. Thus, the signal-to-noise ratio (SNR) for the transmission from AP to relay can be represented as

$$\gamma_0 = \frac{P_1 h_0}{\sigma_0^2}. \tag{1}$$

The corresponding SNR from relay to user k can be obtained as

$$\gamma_k = \frac{P_2 h_k}{\sigma_k^2}. \tag{2}$$

Furthermore, since the data transmission has a latency constraint, the transmission has to adopt finite blocklength (FBL) codes. For the latency-constrained scenario, we assume all the data packets are transmitted or forwarded without additional delay or with a constant delay, so that the total blocklength, which is available for the whole transmission period, can be determined. We denote by M the total blocklength for a transmission period, and respectively by m_0 and m_0' the allocated blocklength to the hops from AP to relay and from relay to multiple users. Namely, we have $m_0 + m_0' = M$. In each transmission period, we denote by D_k the data amount initialized by AP for user $k \in \mathcal{K}$. This implies that on the first hop, the total data amount transmitted from AP to relay node is given by $\sum_{k=1}^{K} D_k$.

Moreover, within FBL regime, there always exists an unignorable decoding error probability, even when the coding rate is below the Shannon capacity. According to [14], with given error probability threshold $\varepsilon_{\max}$, a blocklength m and SNR γ, the maximum coding rate $r_{\max}$ can be characterized as

$$r_{\max} = \mathcal{C}(\gamma) - \sqrt{\frac{V(\gamma)}{m}} Q^{-1}(\varepsilon_{\max}) \log_2 e, \tag{3}$$

where $\mathcal{C}(\gamma) = \log_2(1+\gamma)$ is the Shannon capacity, $V(\gamma) = 1 - \frac{1}{(1+\gamma)^2}$ denotes the channel dispersion and $Q^{-1}(\varepsilon_{\max})$ is the inverse function of Q-function $Q(w) = \int_w^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{w^2}{2}} dw$.

In this work, we also consider a reliability-constrained scenario, where the transmissions on each link have a maximum tolerant error probability $\varepsilon_{\max}$. As a result, for each transmission in the relaying network, the coding rate will be upper-bounded by a corresponding maximum coding rate $r_{\max}$. Different blocklength allocation will definitely affect the maximum allowed coding rate and also the maximum transmitted data amount under the reliability constraint $\varepsilon_{\max}$. Therefore, in the considered reliability-constrained scenario, we aim at maximizing the minimum throughput among all users in the relaying network via blocklength allocation.

In addition, it should be pointed out that different relaying principles at the relay node and different serving strategies for relaying serving multiple users will result in optimization problems in different formulation. More specifically, there are in general two typical relaying principles, i.e., decode-and-forward (DF) relaying and amplify-and-forward (AF) relaying. In DF relaying, the data packet from AP is first decoded at relay. After a successful decoding, the relay will then re-encode and forward the data to the users. In the first hop, it is clearly more beneficial for AP to jointly encoded the data for all users with a large blocklength m_0, since a larger blocklength allows a larger coding rate with given error probability. However, while serving multiple users, as shown in Fig. 2, the relay can choose to either encode and transmit the data to the corresponding user in a time-division multiple access (TDMA) manner (**TDMA-DF**), or encode and broadcast an integrated data packet to all users (**broadcasting-DF**) so that each user has to decode the whole packet to obtain their own data. In AF relaying, the relay will directly enlarge

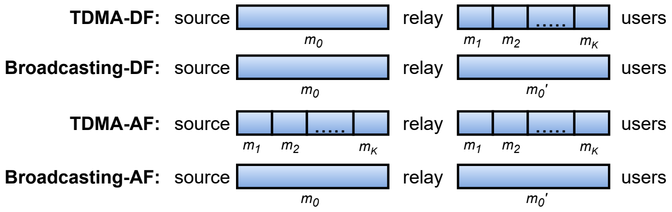

Fig. 2. Examples of four combination strategies.

the received signal from AP and forward to the users. Different from DF relaying, the AF relay is lack of the decoding and encoding capabilities, so that the first hop should have the same blocklength division strategy as the second hop, i.e, both hops either apply an integrated large blocklength or perform in TDMA manner to serve multiple users. Correspondingly, we have for AF relaying, two different cases, **TDMA-AF** and **broadcasting-AF**.

To well investigate the blocklength allocation under different combinations of relaying principles and serving strategies, in the next section, we start with the combination of TDMA-DF strategy, formulate and address the blocklength allocation problem to improve the minimum throughput performance among all users. After that, the solutions will be extended into other combinations in Section IV.

III. BLOCKLENGTH ALLOCATION IN TDMA-DF STRATEGY

A. Problem Formulation

For TDMA-DF strategy, the data for all users are jointly encoded and integrated into one packet in the first hop from AP to relay, i.e., the coding rate for the first hop is $\frac{\sum_{k=1}^{K} D_k}{m_0}$. Under the error probability constraint, we have the coding rate lower than the maximum allowed coding rate, i.e.,

$$\frac{\sum_{k=1}^{K} D_k}{m_0} \leq \mathcal{C}(\gamma_0) - \sqrt{\frac{V(\gamma_0)}{m_0}} Q^{-1}(\varepsilon_{\max}) \log_2 e, \quad (4)$$

which can be reformulated as

$$\sum\nolimits_{k=1}^{K} D_k \leq m_0 \mathcal{C}(\gamma_0) - \sqrt{m_0 V(\gamma_0)} Q^{-1}(\varepsilon_{\max}) \log_2 e \triangleq f_0(m_0). \quad (5)$$

On the other hand, for the second hop, the remaining blocklength m_0' is divided and respectively allocated to different users. We denote by m_k the blocklength allocated to user $k \in \mathcal{K}$ in the second hop. Namely, we have $\sum_{k=1}^{K} m_k = m_0'$. According to the maximum error probability constraint, similar to the first hop, we have

$$D_k \leq m_k \mathcal{C}(\gamma_k) - \sqrt{m_k V(\gamma_k)} Q^{-1}(\varepsilon_{\max}) \log_2 e \triangleq f_k(m_k). \quad (6)$$

Therefore, the problem for maximizing the minimum throughput among all users, i.e., $\min_{k \in \mathcal{K}} D_k$, can be formulated as

$$\text{(P1)} \quad \max_{m_0, \{m_k, D_k\}} \quad \min_{k \in \mathcal{K}} D_k$$

$$\text{s.t.} \quad m_0 + \sum\nolimits_{k=1}^{K} m_k = M, \quad (7)$$

$$\sum\nolimits_{k=1}^{K} D_k \leq f_0(m_0), \quad (8)$$

$$D_k \leq f_k(m_k), \ \forall k \in \mathcal{K}, \quad (9)$$

$$m_0 > 0, \ m_k > 0, \ \forall k \in \mathcal{K}. \quad (10)$$

Algorithm 1 : Iterative Algorithm.

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a) Initialize a feasible point $(m_0^{(0)}, m_k^{(0)}, D_k^{(0)})$ for (P1).
b) Set iteration index $r = 0$.
c) Build convex problem (P1') based on $(m_0^{(r)}, m_k^{(r)}, D_k^{(r)})$.
d) Solve (P2) and get optimal solution $(m_0^{(r*)}, m_k^{(r*)}, D_k^{(r*)})$.
e) If the improvement of $\min_{k \in \mathcal{K}} D_k <$ threshold $\lambda_{th}$
       Return.
   Else
       $(m_0^{(r+1)}, m_k^{(r+1)}, D_k^{(r+1)}) = (m_0^{(r*)}, m_k^{(r*)}, D_k^{(r*)})$.
       $r = r + 1$.
       Back to c).
   End
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However, the functions $f_0(m_0)$ and $f_k(m_k)$ are not concave but convex, so that the problem (P1) is not convex and cannot be directly addressed via convex optimization tool.

B. Iterative Solution

In this subsection, we construct a convex approximation for problem (P1) and accordingly propose an iterative algorithm for an efficient blocklength allocation solution.

Note that $f_i(m_i)$ is convex in m_i, $i \in \{0\} \cup \mathcal{K}$. According to the property of convex function, we have for any given positive value $m_i^{(r)}$

$$f_i(m_i) \geq a_i^{(r)} m_i + b_i^{(r)} \triangleq f_i^{(r)}(m_i), \quad (11)$$

where the constants $a_i(r)$ and $b_i(r)$ are defined as

$$a_i^{(r)} = f_i'(m_i^{(r)}) = \mathcal{C}(\gamma_i) - \frac{1}{2} \sqrt{\frac{V(\gamma_i)}{m_i^{(r)}}} Q^{-1}(\varepsilon_{\max}) \log_2 e, \quad (12)$$

$$b_i^{(r)} = f_i(m_i^{(r)}) - a_i^{(r)} m_i^{(r)}. \quad (13)$$

Clearly, the function $f_i^{(r)}(m_i)$ is affine and also concave, while $f_i(m_i) = f_i^{(r)}(m_i)$ when $m_i = m_i^{(r)}$. By replacing $f_i(m_i)$ with $f_i^{(r)}(m_i)$ in problem (P1), we can obtain a convex approximation (P1')

$$\text{(P1')} \quad \max_{m_0, \{m_k, D_k\}} \quad \min_{k \in \mathcal{K}} D_k$$

$$\text{s.t.} \quad m_0 + \sum\nolimits_{k=1}^{K} m_k = M, \quad (14)$$

$$\sum\nolimits_{k=1}^{K} D_k \leq f_0^{(r)}(m_0), \quad (15)$$

$$D_k \leq f_k^{(r)}(m_k), \ \forall k \in \mathcal{K}, \quad (16)$$

$$m_0 > 0, \ m_k > 0, \ \forall k \in \mathcal{K}. \quad (17)$$

With the assistance of (P1'), we can then propose an iterative algorithm for the solution. After an initialization step ($r = 0$), a feasible local point $(m_0^{(0)}, m_k^{(0)}, D_k^{(0)})$ can be constructed. Then, in the r-th iteration, the convex problem (P1') is established based on $(m_0^{(r)}, m_k^{(r)}, D_k^{(r)})$. After solving (P1'), the obtained optimal point will be adopted as the local point for the next iteration, i.e., $(m_0^{(r+1)}, m_k^{(r+1)}, D_k^{(r+1)})$. By repeating the iterations, the minimum throughput $\min_{k \in \mathcal{K}} D_k$ will be continuously improved and eventually converge to a suboptimal point, which will be the proposed solution for (P1). The algorithm flow is shown below in Algorithm 1.

In the next section, we will respectively build the blocklength allocation problem for the other three strategies, i.e., broadcasting-DF, TDMA-AF and broadcasting-AF. And the iterative algorithm will also be extended.

IV. BLOCKLENGTH ALLOCATION IN OTHER STRATEGIES

A. Broadcasting-DF Strategy

For the broadcasting-DF strategy, both two hops in the relaying network adopt an integrated block for multi-user data transmission, with respectively blocklengths of m_0 and m_0'. Note that different from TDMA-DF strategy, in the second hop of broadcasting-DF strategy, the data for all users are jointly encoded in a single block, such that each user can successfully obtain its own data only when the jointly encoded data packet is correctly decoded. Therefore, in the second hop, under the reliability constraint $\varepsilon_{\max}$, we have

$$\sum_{k=1}^{K} D_k \leq \min_{k \in \mathcal{K}} f_k(m_0'), \tag{18}$$

where the function $f_k(m)$ is defined in the same form as in (6). Since the first hop for broadcasting-DF is the same as that in TDMA-DF, the constraint for the blocklength m_0 in the first hop should remain the same. As a result, the blocklength allocation problem in broadcasting-DF strategy can be formulated as

$$\begin{aligned} \text{(P2)} \max_{m_0, m_0', D_k} \quad & \min_{k \in \mathcal{K}} D_k \\ \text{s.t.} \quad & m_0 + m_0' = M, && (19) \\ & \sum_{k=1}^{K} D_k \leq f_0(m_0), && (20) \\ & \sum_{k=1}^{K} D_k \leq \min_{k \in \mathcal{K}} f_k(m_0'), && (21) \\ & m_0 > 0,\ m_0' > 0. && (22) \end{aligned}$$

Clearly, similar to problem (P1), we have the constraints (20) and (21) not convex, which lead the whole problem (P2) to be nonconvex.

Following the same approach in Section III, we can build the concave function $f_0^{(r)}(m_0)$ and $f_k^{(r)}(m_0')$ based on a local point $(m_0^{(r)}, m_0'^{(r)}, D_k^{(r)})$, as performed in (11). It is also guaranteed that $f_0(m_0) \geq f_0^{(r)}(m_0)$ and $f_k(m_0') \geq f_k^{(r)}(m_0')$ hold, while the equality holds at point $(m_0^{(r)}, m_0'^{(r)}, D_k^{(r)})$. By replacing $f_0(m_0)$ and $f_k(m_0')$ respectively with $f_0^{(r)}(m_0)$ and $f_k^{(r)}(m_0')$, problem (P2) can be converted to a convex one. Then, by adopting the iterative algorithm, the minimum throughput in problem (P2) can be improved by iteratively updating the local point (m_0, m_0', D_k). Finally, the minimum throughput will continuously increase and efficiently converge to a suboptimal point.

B. TDMA-AF Strategy

Different from DF relaying principle, for AF relaying, the second hop should keep the same blocklength allocation and the same coding rate as the first hop, due to the incapability of AF relay in decoding and encoding. For TDMA-AF strategy, we still denote by m_k the allocated blocklength for user k in the second hop, which is also the allocated blocklength for the data oriented to user k in the first hop. Thus, we have $2\sum_{k=1}^{K} m_k = M$.

Since in AF relaying the decoding process only appears at the user side, we should study the SNR for each user to characterize the effects of reliability constraint. The received power at AF relay is in total $P_1 h_0 + \sigma_0^2$, which is then enlarged and send out with power P_2. This indicates that the amplification gain at AF relay is $\frac{P_2}{P_1 h_0 + \sigma_0^2}$. For the user k, the received power is given by $P_2 h_k + \sigma_k^2$, which contains signal power $\frac{P_1 h_0 \cdot P_2}{P_1 h_0 + \sigma_0^2}$, amplified noise power from the first hop $\frac{\sigma_0^2 \cdot P_2}{P_1 h_0 + \sigma_0^2}$ and noise power on the second hop σ_k^2. As a result, the SNR at user k can be represented as

$$\gamma_{\mathrm{AF},k} = \frac{\frac{P_1 h_0 \cdot P_2}{P_1 h_0 + \sigma_0^2}}{\frac{\sigma_0^2 \cdot P_2}{P_1 h_0 + \sigma_0^2} + \sigma_k^2}. \tag{23}$$

We denote by $\varepsilon_{\mathrm{AF},\max}$ the maximum allowed error probability for the decoding process at all users. The throughput D_k for user k during the whole blocklength M is limited by

$$\begin{aligned} D_k \leq\ & m_k \mathcal{C}(\gamma_{\mathrm{AF},k}) - \sqrt{m_k V(\gamma_{\mathrm{AF},k})} Q^{-1}(\varepsilon_{\mathrm{AF},\max}) \log_2 e \\ \triangleq\ & f_{\mathrm{AF},k}(m_k). \end{aligned} \tag{24}$$

To sum up, the blocklength allocation problem for maximizing the minimum throughput under TDMA-AF strategy is formulated as

$$\begin{aligned} \text{(P3)} \max_{\{m_k, D_k\}} \quad & \min_{k \in \mathcal{K}} D_k \\ \text{s.t.} \quad & 2\sum_{k=1}^{K} m_k = M, && (25) \\ & D_k \leq f_{\mathrm{AF},k}(m_k),\ \forall k \in \mathcal{K}, && (26) \\ & m_k > 0,\ \forall k \in \mathcal{K}. && (27) \end{aligned}$$

Then, similar to the approach in DF relaying, we can define a concave approximation $f_{\mathrm{AF},k}^{(r)}(m_k)$ for $f_{\mathrm{AF},k}(m_k)$, as

$$f_{\mathrm{AF},k}(m_i) \geq a_{\mathrm{AF},k}^{(r)} m_i + b_{\mathrm{AF},k}^{(r)} \triangleq f_{\mathrm{AF},k}^{(r)}(m_i), \tag{28}$$

where the constants $a_{\mathrm{AF},k}(r)$ and $b_{\mathrm{AF},k}(r)$ are defined as

$$a_{\mathrm{AF},k}^{(r)} = \mathcal{C}(\gamma_{\mathrm{AF},k}) - \frac{1}{2}\sqrt{\frac{V(\gamma_{\mathrm{AF},k})}{m_k^{(r)}}} Q^{-1}(\varepsilon_{\mathrm{AF},\max}) \log_2 e, \tag{29}$$

$$b_{\mathrm{AF},k}^{(r)} = f_{\mathrm{AF},k}(m_k^{(r)}) - a_{\mathrm{AF},k}^{(r)} m_k^{(r)}. \tag{30}$$

The approximation is clearly tight when $m_k = m_k^{(r)}$. Based on the approximation, we can construct an iterative algorithm. In each iteration r, we convert the problem (P3) to a convex one on a local point $(m_k^{(r)}, D_k^{(r)})$ via applying the concave approximation $f_{\mathrm{AF},k}^{(r)}(m_k)$. By repeating the iterations and constantly updating the local point, the objective will eventually converge to a suboptimal point.

C. Broadcasting-AF Strategy

For broadcasting-AF strategy, similar to TDMA-AF strategy, the two hops in the relaying should keep the same blocklength and the same coding rate, i.e., $m_0 = m_0'$. With total blocklength defined as M, the optimal blocklength allocation will be $m_0 = m_0' = \frac{M}{2}$ since a larger blocklength will be beneficial in providing a larger coding rate in FBL regime. Under the reliability constraint $\varepsilon_{\mathrm{AF},\max}$ for

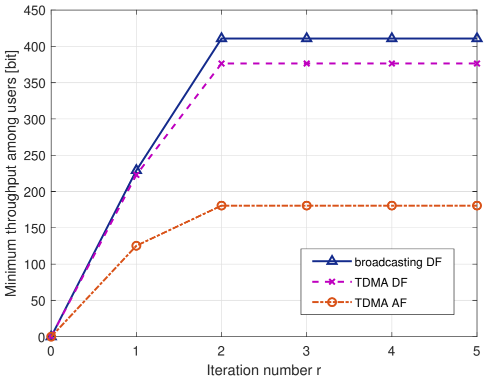

Fig. 3. Convergence behaviour of proposed iterative algorithms.

AF relaying, the sum throughput over all users is limited by $\sum_{k=1}^{K} D_k \leq \min_{k\in\mathcal{K}} f_{\mathrm{AF},k}(\frac{M}{2})$. Hence, the minimum throughput among all users can be optimally maximized as $\min_{k\in\mathcal{K}} D_k = \frac{1}{K}\min_{k\in\mathcal{K}} f_{\mathrm{AF},k}(\frac{M}{2})$.

V. SIMULATION RESULTS

In this section, via simulations, we validate the proposed optimization algorithms for different strategies and compare the maximized minimum achievable throughput with different setups. In simulations, we consider a 2D topology with the source node placed at position $(0,0)$km and a relay node at $(1,0)$km. A group of $K=5$ users are deployed in a small area around position $(2,0)$km. Note that we assume that the area is small such that the distance from relay to all users can be considered as the same, i.e., 1km. By default, we define the transmit power as $P_1 = P_2 = 4$W, noise power $\sigma_0^2 = \sigma_k^2 = -70$dBm and total blocklength $M = 2000$. The channel gains h_i $(\forall i \in \mathcal{K}\cup\{0\})$ is defined as $h_i = \frac{\beta}{h_i^{\alpha}}$, where $\beta = -20$dB is the reference gain at reference distance 1m, d_i is the distance of corresponding links and $\alpha = 2.67$ is the path loss exponent factor. In addition, we set the error probability limit in DF relaying as $\varepsilon_{\max} = 10^{-9}$. Since AF relaying principle contains only one decoding process, for a fairness in comparison, we set the maximum allowed error probability in AF relaying as $\varepsilon_{\mathrm{AF},\max} = 1-(1-\varepsilon_{\max})^2 = 2\varepsilon_{\max} - \varepsilon_{\max}^2$. Note that although we start with the scenario where all users have the same channel gains to relay to facilitate the analysis of different strategies, we will also investigate the behaviours in fading channels later.

At first, we show in Fig. 3 the convergence behaviour of the proposed iterative algorithm for the strategies of TDMA-DF, broadcasting-DF and TDMA-AF. As depicted in Fig. 3, the minimum throughput in all iteratvie algorithms increases in the iteration number r, and the throughput performance converges within 2 iterations, which implies a high efficiency of our proposed algorithms.

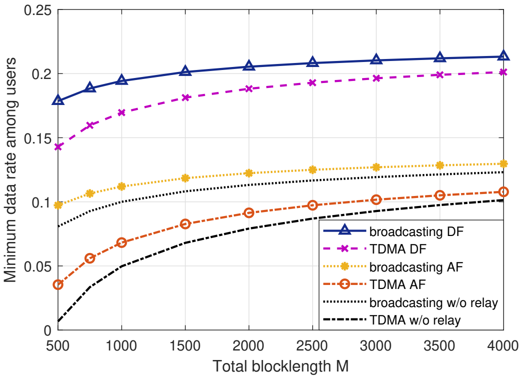

Fig. 4. Achievable minimum data rate among users with respect to different total blocklength under different strategies.

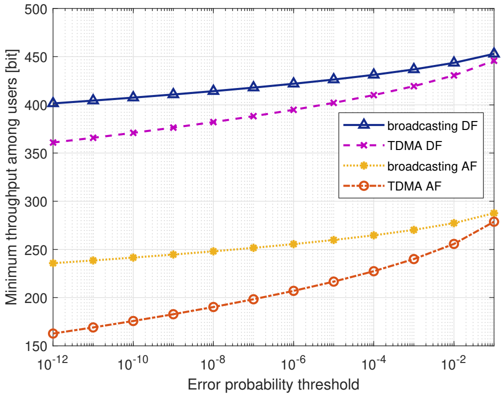

Fig. 5. Minimum achievable throughput among users with respect to reliability constraint under different strategies.

Then, in Fig. 4 we compare the throughput performance of all the four strategies, i.e., TDMA-DF, broadcasting-DF, TDMA-AF and broadcasting-AF, by evaluating the minimum data rate among users, i.e., $\frac{\min_{k\in\mathcal{K}} D_k}{M}$, with varying total blocklength M. As shown in Fig. 4, a larger total blocklength, i.e., more block resources, will lead to a higher data rate for all strategies. Furthermore, for both the broadcasting and TDMA strategies, DF relaying generally outperforms AF relaying. Note that by default, we have set up the same channel quality for all users. In the default homogeneous scenario, the broadcasting strategy shows a better performance than TDMA for both DF and AF relaying, which indicates the advantages of decoding with a larger blocklength. And the benefits of broadcasting strategy over TDMA are shown to recede when the total blocklength M increases. In addition, we also show in Fig.4 the performance of direct transmission. Via comparison, it is also observed that with the same downlink strategy, either broadcasting or TDMA, both DF and AF relaying are capable of enhancing the throughput, which indicates the benefits of relaying in FBL regime. Furthermore, we also evaluate the minimum achievable throughput among all users under reliability constraint with varying maximum allowed error probability under different strategies. From the results shown in Fig. 5, we can observe that DF relaying still outperforms AF relaying and clearly a looser reliability constraint will lead to a larger throughput in the multi-user relaying network. We also notice that in the homogeneous scenario, the performance gap of broadcasting over TDMA for both DF and AF relaying is reduced when the maximum error probability becomes larger. This is due to that with larger $\varepsilon_{\max}$, the reliability constraint on each block in TDMA is relaxed, so that TDMA has more freedom in improving the throughput.

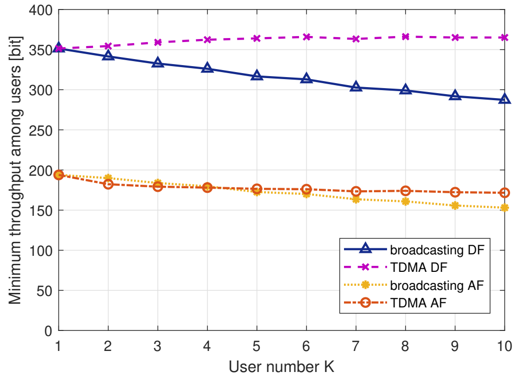

Fig. 6. Average minimum achievable throughput among users over 400 fading realizations.

Afterwards, we finalize our simulation evaluation by studying the scenario with fading channels. We update all the channel gains as $h_i = \frac{\beta z_i}{d_i^{\alpha}}$, where z_i is the Rayleigh fading variable with scale factor of 1. Next we evaluate all four strategies under 400 fading realizations and display the average throughput performance in Fig. 6 with different user number K and $M = 400K$. Since broadcasting strategies is restricted by the bottleneck user, more served users will generally introduce a higher variance in channel gains such that as shown in Fig. 6 the average throughput performance decreases for both broadcasting DF and AF. By contrast, TDMA strategy is observed to be more robust with respect to the channel differences. It should be pointed out that we have also introduced fading channel for the link from source to relay. For TDMA-DF, the deep fading on the first hop can be compensated via allocating more blocklengths, while the possible advantages of high channel quality can also be deployed to saving more blocklengths for the second hop. As a result, it is shown in Fig. 6 that TDMA-DF can even lead to a better performance with more served users. However, for TDMA-AF, both hops should keep the same blocklengths allocations for all users, which results in a declining performance as the user number increases.

VI. CONCLUSION

In this paper, we have focused on a multi-user relaying network and studied the four combinations of relaying principles and downlink strategies, i.e., TDMA-DF, broadcasting-DF, TDMA-AF and broadcasting-AF strategies.Under a reliability constraint, we have formulated optimization problems for each combination strategy, to maximize the minimum throughput among all users via optimizing the blocklength allocation. To address the nonconvex problems, we first propose an iterative algorithm for TDMA-DF strategy, which is then extended to other combination strategies, i.e., broadcasting-DF and TDMA-AF, while the optimal blocklength allocation for broadcasting-AF strategy can be directly found via analysis. Finally, the simulation results confirm the convergence behaviours of our proposed iterative algorithms, and show the performance advantages of DF relaying over AF relaying, as well as the benefits of deploying relay in the multi-user network. Via simulations, we also find that the performance of broadcasting strategy is generally limited by the user with worst channel. Meanwhile, the TDMA strategy, especially the TDMA-DF, can compensate the lower channel gains, exploit the larger channel gains through blocklength allocation and outperforms when the channel qualities for all users have relatively larger differences.

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